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APPLICATION OF PERFORMANCE CONCEPT IN EVALUATION, SPECIFICATION, AND SELECTION OF ROOFING MATERIALS

Roger L. Bonafont
Ruberoid Limited, Enfield, Middlesex, England

FOREWORD

Perhaps the most unsatisfactory aspect of membrane roofing technology today is the absence of a clear rationale by means of which roofing materials and systems may be evaluated on a common bases and selected to meet realistic requirements with adequate safety margins.

The paper is an attempt at the presentation of a rationale, if only to stimulate discussion and possibly to suggest further lines of research.

Part I deals primarily with the conceptual aspects of performance, highlighting a pathological approach to roofing problems as a basis for design and specification.

Part II summarizes views on the structural aspects of membrane systems and develops the concept of structural durability as an extension of fatigue endurance.

PART I
APPLICATION OF PERFORMANCE CONCEPT IN EVALUATION, SPECIFICATION AND SELECTION OF ROOFING MATERIALS

General Considerations

1. Introduction

The performance concept presupposes that performance can be quantified, at least approximately. Application of the concept also presupposes that measured or calculated performance under given conditions can be related to long-term durability under service conditions. The prime object of the concept is to reduce the risk of premature or unanticipated call-back because of malfunction of the system.

The practical utility of the performance concept depends upon the following prerequisites:

(1) that performance can be defined unequivocally by measured or calculated quantity.

(2) that phenomenological processes and related failure modes associated with performance can be adequately described by means of idealised models. The models need to embrace all relevant system variables in functional relationships that provide a tolerably accurate and workable theoretical framework. The utility of any idealised model is judged solely by its ability to predict observable behavior.

(3) that there be common industry agreement on safety factors or margins of performance in excess of stated requirements.

2. Key Words

The building owner is primarily concerned with the **performance** of the membrane **system** as a whole. Thus materials, products and system components cannot be judged in isolation, but only in combination with one another.

Key words are:

(1) **the system,** and

(2) the **performance** achievable with that system.

3. The System

A roofing system includes some or all of the following components:

(1) structural deck

(2) vapor barrier, if any

(3) thermal insulation, if any

(4) membrane (composed of one or more preformed sheetings bonded together or otherwise sealed to form a single composite waterproof membrane, or may be composed entirely of liquid applied material with or without reinforcement)

(5) membrane protection or surfacing treatment, if any.

The design or system variables of interest are those related to:

(a) the physical dimensions of elements and sub-elements
(b) the properties of materials in each element at any given time.
(c) the combination and relative position of elements within the system; because combination will itself influence performance (e.g. warm roof, cold roof and inverted roof assembly).
(d) workmanship, which has an indirect bearing on the design variables in that it determines the degree to which actual dimensions and possibly properties correspond to those foreseen at the design stage.

4. General Definition of System Performance

The function of performance is to ensure:

(1) fitness for a specified purpose when the system is properly installed, and
(2) ability to continue to perform as required for an acceptable or specified period of time.

System design and related materials selection must satisfy two distinct sets of requirements:

- **short-term** requirements insuring that system components and assembly operations are reasonably tolerant to site and weather conditions.
- **long-term** performance under building service conditions.

The specifier is mainly concerned with the long-term requirements, assuming that the component manufacturers have met the practical short term contractor's requirements. For obvious reasons, however, both sets of requirements are of equal importance.

Any laboratory evaluation program must assess compliance with both long and short-term requirements. Experience in England has tended to show that single-layer preformed sheeting systems and liquid-applied membranes involve a greater risk of call-back and are not favored in some quarters for this reason. The greater risk is in a large measure due to the lesser tolerance of such systems to site and weather conditions in England.

5. An Approach to Design Methodology

The design and specification procedure must of course deal with matters other than avoiding failure. A system is required to comply with building regulations, and may have to meet specific requirements in respect of fire and safety. These aspects are not the subject of this paper, but must not be overlooked in the overall design process.

All cases of roofing failure (other than those caused by natural cataclysms) can be traced to human errors which can be classified as follows:

(a) **Design Errors** resulting from ignorance or negligence.
(b) **Workmanship defects,** or misuse or abuse of materials on site, often aggravated by insufficient product application tolerances to accommodate short-term requirements.
(c) **Misjudgements** concerning the level of performance required in a particular case. This is a failure, despite sound details and good workmanship. It can result from failure to anticipate the appropriate margin of safety. Or it may spring from a deliberate gamble to cut initial cost.

The performance concept can only be of assistance with regard to improvement in the quality of judgements. Errors arising from (a) and (b) can only be dealt with by education, training and supervision of those involved in design and construction at all levels.

Because of design imponderables, material tolerances, and workmanship variability, a system must offer more than is strictly required of it to give a high probability of meeting the performance requirements under service conditions.

The probable consequence of a misjudgment is a roofing failure. A useful starting point for a design methodology is to identify and list failures according to the type of visible damage to the roofing system, if only to ensure that attention is directed to the types of malfunction likely to occur in practice.

The more common types of waterproofing membrane failure that occur under service conditions are shown in figure 1.

The following comments do not purport to deal with the relevant design aspects but hopefully serve to set a scene.

5.1 Wind Damage

Wind damage mainly affects self-finished membranes or those under light weight surface treatments.

In the British Isles, wind loads for design purposes are calculated in accordance with the Code of Practice CP3: Chapter V; Part 2: 1972. The commonly accepted design procedures assume that wind effects are equivalent to static loads of the specified magnitude. However, it must not be overlooked that the action of wind is complex, infinitely variable in its local effects, a dynamic loading phenomenon in which the inertia of the roof system may sometimes have an important influence.

Wind action may cause cumulative, unseen weakening of the roof system over a period of time such that actual damage appears to occur during a relatively light storm. The correlation between intensity of storm and related damage is therefore frequently confusing.

Existing engineering knowledge is crudely sufficient to deal with wind loads by the application of structural design principles, but the complexity of the problem is such that a conservative design approach is desirable.

Properties required for wind-uplift resistance are:

- Laminar strength under repeated loads. (The weakest link is frequently the substrate to which the membrane is bonded.)
- Peel strength between the membrane and the substrate and/or interply peel strength.
- Nail holding strength (the lesser of nail pull-out or nail head pull-through).

5.2 **Slippage of Membrane** on slopes or at upstands.

Slippage is prevented by choosing adhesives with resistance to creep at the highest service temperature likely to occur on the roof. The incidence of slippage has increased in recent years because of increasing thermal insulation in roof construction.

In Britain and on roofs with a pitch in excess of 25°, it is usual to use 115/15 bitumen to bond built-up membranes. When selecting or specifiying hot melt adhesive, it is important to ensure by screening tests that their rheological properties are not degraded by normal heating in kettles on site.

Alternatively, membranes must be mechanically anchored to resist movement down the slope. The roofings so anchored must develop sufficient resistance at anchorages to avoid tearing or deforming excessively, and the anchors must themselves be able to carry the loads imposed upon them.

The simplest anchorage system consists of galvanized or treated roofing nails driven at 50 mm centres into timber battens set within the thickness of the thermal insulation beneath.

In traditional built-up membrane systems, the base fabric of the bituminous roofing must be strong enough to resist tearing around roofing nail shanks. At elevated temperature, bitumen provides little, if any, permanent additional resistance at nail anchorages.

The need for mechanical anchorage must clearly be anticipated at the design stage to provide for nailing battens or other forms of mechanical anchorage.

The system parameters to design in this respect are:

- Thermal stability of hot melt adhesive (susceptibility to fall-back).
- Shear-flow properties of adhesives at highest in-use roof surface temperature.
- Tear resistance of felt base around roof nail shanks or other fastening devices.

5.3 **Cosmetic Deficiencies**

Visual defects in the membrane may remain as aesthetic blemishes of little major consequence, or they may portend more serious effects likely to result in membrane degradation.

The more common visual defects are:

- lack of color fastness
- self-generated staining
- corrosion or pollution effects
- loss of surfacing granules
- flaking or stripping of bituminous coatings
- membrane blisters: from substrate: interply or in surface coatings
- wrinkling
- surface crazing or alligatoring
- generalized surface disintegration (as indicative of lack of inherent durability)

The avoidance of cosmetic deficiencies depends upon the informed selection of materials based on experience or when experience is not available, on simulated weathering or other accelerated exposure tests. It should be noted that simulated tests do not always provide a reliable indication of probable cosmetic effects under service conditions.

5.4 **Water Penetration:** presumed through the roof covering in the absence of obvious defects from (5.1), (5.2), and (5.3) above.

Cases of suspected water penetration fall into two categories, (1) **Apparent Penetration** and (2) **Actual Penetration.** It is important to appreciate that the appearance of water on the underside of a roof construction does not necessarily imply a leak through the roof covering.

5.4.1 **Apparent Penetration**

The more common causes are:

- •Condensation.
- •Water entrapped within the roof system during construction (e.g. residual water from wet construction process or rain).

The subject of condensation within roofs has received a great deal of attention in the last twenty years or so. There is ample documentation on the subject in most countries and little excuse today for failure to deal with potential problems at the design stage.

The finer points of the subject have now been studied in depth and are well documented. In Britain, the Department of the Environment and the Building Research Establishment have produced many informative books and technical papers. Advanced research into moisture movement within roof and wall systems has been undertaken at the Lund Institute of Technology, Sweden.

5.4.2 **Actual Penetration**

Penetration of rainwater through a roof covering arises because of an unintended and generally localized discontinuity in the membrane.

Structural discontinuities are themselves conveniently classified into three categories:

5.4.2.1 **Self Induced Delamination:** a separation or disjoining of previously bonded preformed elements of the covering. This is a rare type of failure that affected certain elastomeric sheets bonded with an unsuitable adhesive. It is therefore a possible mode of failure.

5.4.2.2 **Puncturing** of the covering by:

- •Human action: e.g. carelessness, abuse, accident or excessive roof traffic
- •Animal action: e.g. birds, termites
- •Natural action: e.g. hail stones

Puncture resistance of a given membrane system is a property which is difficult to quantify in an unequivocal manner. Resistance to point loads is highly dependent upon the test procedure and on the mechanical properties of the substrate beneath the membrane. There is often a poor correlation between the results from one type of test with those from another.

Membrane systems can nonetheless be generally ranked in order of resistance to penetration, but the problem of deciding how much resistance is required in any particular case remains largely a matter of judgement.

System tests should include:

(a) Resistance to penetration at elevated temperature under a static load.

(b) Impact resistance at low temperature.

5.4.2.3 **Structural Rupture**

Rupture of the membrane by in-plane forces is caused by the local over-stressing of the membrane with or without prior fatigue weakening.

Membrane stresses are generated primarily by movements of materials in intimate contact with the membrane and therefore able to deform it.

Ice on flat or ponded roofs should be viewed as a substrate capable of generating stresses in the membrane so covered. Caked mud deposits on a membrane may also generate stresses capable of causing rupture.

In exceptional cases, pronounced shrinkage or thermal stresses in a restrained membrane may contribute to its rupture.

The general case of substrate movement is advantageously divided into membrane stresses generated by (a) an **irreversible movement** where the cold flow properties of the membrane may have a mitigating effect and (b) **reversible movement** where the cold flow properties may exacerbate membrane endurance and give rise to fatigue weakening of the membrane leading to early rupture.

It does not follow that a system able to accommodate large irreversible movements will necessarily provide correspondingly high fatigue endurance under repeated loading cycles.

Avoidance of membrane rupture is an important design matter to be dealt with by one or more of the following:

(1) Judicious selection of dimensionally stable substrate systems to minimize movement and/or rate of movement at joints over other critical zones,

(2) Isolation, as far as possible, of the membrane from substrate movement by loose laying or partial bonding techniques (subject to ballasting or other measures to deal with wind), and

(3) Application of structural principles to the design of the membrane itself.

Part II of this report deals more specifically with the structural aspects of membrane systems.

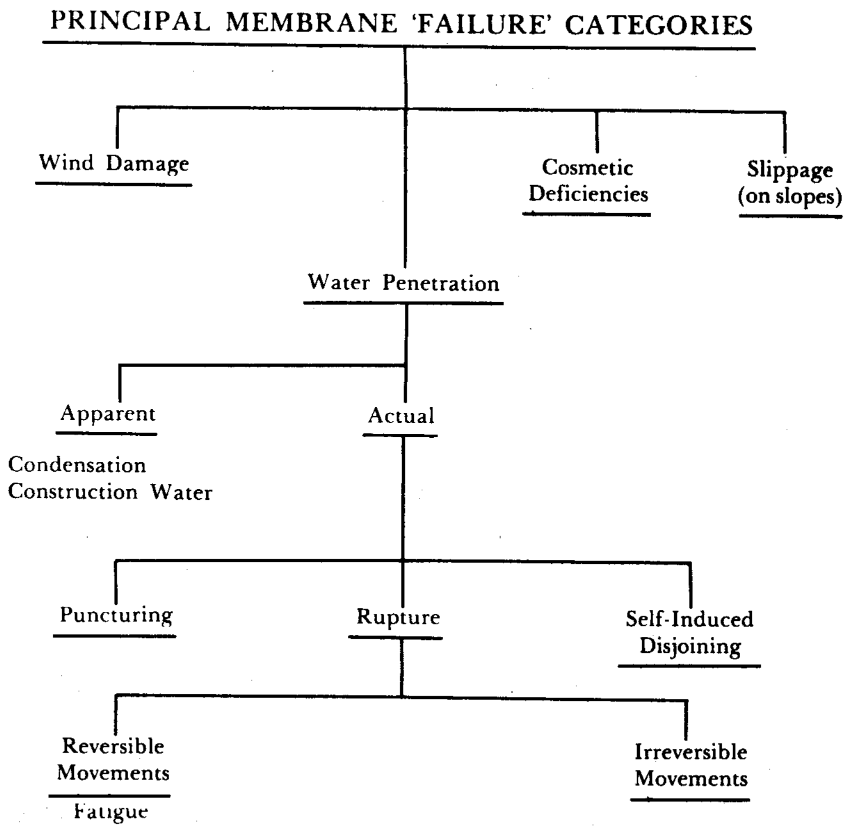

FIGURE 1

PART I

PART II

STRUCTURAL ASPECTS OF MEMBRANE SYSTEMS

INTRODUCTION

Inherent durability of materials – i.e., resistance to weathering and aging – differs from **structural durability** – their resistance to stress. As membranes age, however, they lose strength. The design process must allow for this perennial loss in structural durability.

I advance the following hypothesis:

Given correct design, detailing, sound workmanship, and materials with inherent durability to weathering and natural aging, a membrane system will remain waterproof indefinitely if maintained at all times in a stress-free state.

As no substrate supporting a waterproof membrane system can be guaranteed entirely free of movement during the life of a building, and since it is virtually impossible to isolate the membrane from contiguous construction elements which restrain or then distort the membrane, all waterproofing membranes require structural durability.

Since the bituminous membrane system remains today by far the most widely used everywhere, I shall focus on the structural behavior of such systems.

The idealized membrane structural model (described elsewhere)[1] applies to reinforced bituminous systems, but may be readily adapted to systems composed either wholly or partly of preformed elastomeric sheets.

The mechanical action of a bituminous system is complicated by the visco-elastic nature of bitumen. (The rheological properties of bitumen have been well documented, notably by the many informative publications prepared by the Konoklijke Shell Laboratorium, Amsterdam, and will therefore not be discussed in detail here.)

Bitumen is a linear visco-elastic material of the Boltzmann type. For this reason and with the object of simplifying some of the mathematical relationships to follow, equations will be written where appropriate in their operational form using an asterik* to signify an image function, $f^*(p)$, obtained by the Carson-Laplace transform of the corresponding object function $f(t)$. The transformation is with respect to the time variable t. The operational form of the product of two or more time dependent object functions, g, f, etc. will be shown symbolically by:

$$\{g f.....\}^*$$

A list of symbols is given in Appendix I.

BITUMEN RHEOLOGY AND AGING CHARACTERISTICS

The constant load flow function for bitumen at constant temperature is given by:

(1) $$F(t) = J + t/3_\eta \; t^\alpha/K$$

where J = an instantaneous elastic compliance
$t/3\eta$ = Newtonian viscous flow component
t^α/K = delayed elastic deformation component
and t = elapsed time measured from application of a constant load

The reciprocal of $F(t)$ is known as the constant-load stiffness $S(t)$, discussed in relevant Shell publications.

The parameters J, η, K and α are calculable for any normal blown bitumen from the following input data:

(i) Temperature at which penetration of bitumen is 800 dmm (usually close to the R & B softening point)
(ii) penetration in dmm at some other temperature, say 25 °C
(iii) The actual constant temperature of interest for which the parameters are required.

A "normal" bitumen gives a linear relationship between temperature and logarithm of penetration at least up to the R & B softening point temperature.

For many practical problems, notably those where the combination of loading time and temperature is such that the constant load stiffness is between 1 and 1000 kgf/cm^2, the delayed elastic component of deformation predominates and the corresponding flow function at constant temperature may be taken more simply and with sufficient accuracy as:

(2) $F(t) = t^{\alpha}/K$

A rheological equation of a material expresses the intrinsic relationship between deformation and the stress needed to produce that deformation. The generalized rheological equation for a bitumen whose steady-state flow function corresponds to (2) above approximates to the ideal Boltzmann relationship given by:

(3) $\tau^* = 1/3\ r^* \{K\gamma\}^*$

where γ is the shear distortion produced by the shear stress τ.

The relaxation function r(t) is related to the 'memory' or visco elastic characteristic of the bitumen and is given by:

(4) $$r(t) = \frac{t^{-\alpha}\sin\pi\alpha}{\pi\alpha}$$

Its transform is:

(5) $$r^*(p) = \frac{p^{\alpha}}{P(1+\alpha)}$$

In the most general case, the parameter K is to be regarded as a time dependent function determined by the temperature history and the age hardening of the bitumen. Both temperature changes and age-hardening affect bitumen penetration, and therefore K.

The aging of bitumen protected from the direct effects of solar radiation is primarily the result of loss of volatiles and oxidation. The hardening of bitumen manifests itself by a progressive rise in R & B softening point and a corresponding fall in penetration at standard temperature. The inherent waterproofing properties of bitumen do not appear to be significantly affected by hardening itself.

A three-year program of field work in Europe by the nine members of the Ruberoid Intercompany Technical Committee (RITC members given in Appendix II) has shown that:

(i) There tends to be a progressive hardening of bitumen that follows an approximately exponential law. (For practical purposes, terminal hardness is reached within 15 years.)

(ii) The expected terminal increase in R&B sofening point is about 30%.

(iii) The expected terminal decrease in Penetration at 25°C is about 50%.

Thus, a 105/35 bitumen tends to harden to a 137/17 bitumen within 15 years, and its K-value at 10°C increases from about 70 to 250 kgf/cm^2.

The results of the survey show considerable scatter and the terminal values (ii) and (iii) can only be looked upon as the probable changes. A statistical analysis to investigate the effects of thermal resistance and type of felt used showed no significant differences in the degree of hardening.

By contrast, the parameter α does not appear to be significantly affected by aging or by temperature changes in the range of normal roof service temperatures.

DIFFERENTIAL EQUATION FOR MEMBRANE DISPLACEMENT (ONE DIMENSIONAL CASE)

The idealized system model is shown in Figure 1.

It is convenient to begin with a one dimensional single layer case where displacements and stresses are characterised by a single spatial dimension, x, with the origin at the centre line of a substrate opening.

The model comprises four basic construction elements, but the model may often be simplified when adapted to particular cases of practical interest.

The basic elements of the single layer system are:

(i) **the elastic (waterproofing) membrane:** In the case of traditional preformed bituminous sheetings, the membrane is the reinforcing fabric whose structural integrity is vital to the waterproofing characteristics of the system, because it holds together the bituminous coating needed to assure impermeability.

(ii) **the adhesive layer:** of thickness, h, which connects the membrane to the substrate in a bonded membrane system. Differential movements between membrane and substrate produce shear effects in the adhesive layer, which in turn generate membrane traction.

(iii) **the substrate:** i.e., support for the bonded membrane. In warm-roof construction, the substrate is the thermal insulation. The substrate is viewed in the model as an idealized elastic material with a finite shear rigidity M, and is introduced to show later the effect on membrane stress of substrate rigidity.

(iv) **the substructure:** i.e., the load-bearing part of the roof construction, which may be regarded as stiff by comparison with the other three superimposed elements. (If insulation is omitted, the deck is the substrate.)

The severest condition of membrane strain occurs when the membrane is firmly bonded to its supporting substrate such that movements in the substrate are directly transferred without slip across the adhesive layer/substrate interface.

Membrane strains will peak in a relatively narrow zone immediately above a substrate joint or fissure where movements on either side may occur simultaneously and in opposite directions to produce a variable opening. Such local movement will, sooner or later, cause a fissure to develop just within the adhesive layer, and repeated or sustained movement will cause the nascent fissure to extend upwards from the substrate towards the upper surface of the membrane, unless otherwise arrested by some effective form of reinforcement or membrane.

The mathematical model is constructed to represent the situation described above. By considering the equilibrium of forces and the compatibility of deformations, including unrestrained thermal movements, in a small element of the system, one can show that the displacement, u, of any arbitrary material point on the membrane must satisfy the following operational differential equation:

(6) $(3hEM)u^*_{xx} + Er^* \{Ku_{xx}\}^* - r^*M\{Ku\}^* = - Mr^* \{K(\nu + \bar{w})\}^*$

where u_{xx} is short for $\partial^2 u/\partial x^2$

At constant temperature, with no age-hardening effects, the stiffness parameter K is sensibly constant. Eq(6) then simplifies to:

(7) $u^*_{xx} - a^2u^* = a^2\nu^*$

(8) where $a^2 = \dfrac{r^*KM}{E(r^*K + 3hM)}$

A finite substrate rigidity M, increases the thickness of adhesive layer, h, to an apparent thickness of hr^*K/M, which in turn reduces the general level of membrane stress accordingly. Infinite substrate flexibility virtually isolates the membrane from movements beneath.

The membrane strain, ϵ, is given by the displacement differential:

(9) $\epsilon^* = u^*_x - \bar{u}^*_x$

Solving Eq(6)

The solution of equations (6) and (7) depends on defining proper boundary conditions for the differential equations. This task is best achieved by locating the origin of the co-ordinate system at, say, a joint in the substrate. A second boundary is chosen preferably at point of symmetry, such that the deformed system beyond the second boundary is a mirror image of the system immediately before it. The distance between the origin and the far boundary is taken as L, which may be taken as large as desired. Failure to fix a second boundary at initially finite distance L can lead to solutions which are physically incompatible with the dimensional requirements of a real system.

The corresponding boundary conditions for the membrane displacement in the case of a continuous and unbroken membrane are:

$$u(L) - u(o) = o$$

and these two conditions enable the solution of (6), and the special case (7), to be fully determined in terms of deck movement and of thermal effects, if any.

UNRESTRAINED THERMAL MOVEMENT

Unrestrained thermal movements require the membrane and substrate to be viewed as cut at an axis of symmetry.

It is convenient to choose the origin, x = o, for the position of the cut line. A temperature drop of, say, θ°C will then produce a thermal movement to the right of the origin towards the second axis of symmetry at x = L.

The unrestrained thermal movements will then be:

For the membrane:

(10) $\bar{u} = \theta k_m(L - x)$

For any point at substrate/bonding layer interface.

(11) $\bar{w} = \theta k_s(L - x)$

Where k_m and k_s are the coefficients of thermal expansion for membrane and substrate interface respectively.

MOVEMENT CAPABILITY OF MEMBRANE

In order to compare under standard conditions the movement capability and/or fatigue endurance of a particular combination of membrane and adhesive layer elements (i) and (ii), it is sufficient for the purpose to assume a standard substrate movement of the general form:

$\bar{w} = ½g(1 - x/L)$ with $\nu = o$

where g is a function of time only and is equal to the joint opening at the junction of two substrate elements (at x = o).

The solution if Eq (6) gives the following expression for maximum membrane strain at x = o for large values of L:

(12) $\epsilon_o^* = \frac{1}{2}\sqrt{r^*/3hE}\{\sqrt{K}g\}^* + k_m\theta^*$

The first term is the strain produced by a fissure or joint movement in the substrate and the second term is the strain produced by a temperature drop which would otherwise cause the membrane element (i) to contract.

Note that the adhesive layer stiffness coefficient, K, is assumed to be a time dependent function, to make provision for age hardening effects in that layer.

In a laboratory test at steady temperature, the magnitude of K is sensibly constant, and θ = o, in which case Eq (12) reduces to:

(12a) $\epsilon_o^* = \frac{1}{2}g^*\sqrt{r^*K/3hE}$

PROPOSED STANDARD CONDITIONS FOR NORMALISED MOVEMENT AND AGING EFFECTS

The fatigue endurance of a membrane system may be arbitrarily but nevertheless usefully defined for a standard diurnal movement condition with allowance for an expected amount of age hardening in the adhesive layer.

The standard diurnal movement may be taken as:

(13) $g(t) = \frac{1}{2}g_o(1-\cos\omega t)$

where $\omega = 2\pi/T$

and T = diurnal period in seconds.

Expected age hardening effects in traditional bituminous roofing systems according to RITC surveys appear to follow approximately the following:

(14) $\sqrt{K} \simeq H - \Delta H \exp(-\mu t)$

The ratio μ/ω is approximately 0.000145.

Values of H and ΔH may be estimated given the R&B softening point and penetration of the grade of blown bitumen employed in the system, and a standard temperature, possibly equal to the average annual air temperature.

The solution of Eq (12a) subject to the conditions above results in a periodic strain function whose envelope of tensile peak is given:

(15) $$\hat{\epsilon}(n) = \beta g_o\sqrt{K(n)}\,\hat{Z}(n)$$

where $\beta = 1/4\sqrt{3hE}$

$$\sqrt{K(n)} = H - \Delta H \exp(-\mu nT)$$

$$\hat{Z} = A(1 + B/n^{\frac{1}{2}\alpha})$$

$$A = \text{a constant} = \sqrt{(2\pi)^\alpha/\Gamma(1+\alpha)T^\alpha}$$

$$B = \text{a constant} = 1/\Gamma(1-\tfrac{1}{2}\alpha)(2\pi)^{\frac{1}{2}\alpha}$$

n = n^{th} movement cycle

An inspection of Eq (15) shows that the peak membrane strain is not generally constant from one cycle to the next. Indeed, there are two opposing effects that cause peak strain to vary. The first is the adhesive layer's visco-elastic response, which tends to reduce the magnitude of the peak tensile strain in each successive cycle towards a lower limit. The second is the age-hardening, which increases strains because of the adhesive layer's greater rigidity. The two effects occur simultaneously although the visco-elastic phenomenon predominates in the early stages, while hardening becomes of increasing importance in middle and later phases of useful life.

The peak membrane strain tends towards a terminal plateau of:

(15a) $$\hat{\epsilon}(\infty) = \beta g_o HA$$

FATIGUE CHARACTERISTICS OF THE MEMBRANE

The limit of fatigue endurance of a membrane system is reached when movements of the substrate cause a visible fissure to appear in the uppermost surface of the system. When this occurs, it is reasonable to infer that the entire thickness of the membrane has ruptured, or that its continuity and therefore impermeability have been permanently weakened to an extent sufficient to compromise its utility.

The fatigue endurance of a given system may be quantified by the **number of cycles** to attain the limit state defined above. The endurance will clearly depend upon environmental conditions, but the endurance of systems may at least be compared against standard conditions, and ranked accordingly.

A visible fatigue crack in the exposed surface of a bituminous membrane may arise in one of two ways:

- from the local rupture of the reinforcement, or
- a premature fatigue rupture of the bituminous coating itself (probably as a consequence of lack of stiffness in the reinforcement which allows excessive strains to develop in the coating).

It is possible to cause a fissure through the entire mass of a bituminous membrane system without of necessity rupturing any of the reinforcing fabrics embedded within it.

Both bitumen and traditional felt reinforcements suffer from the weakening effects of repeated loadings, and may be ruptured at comparatively small strains, if these are applied a sufficiently large number of times.

The fatigue characteristic is primarily a statement about the relationship between strain and the corresponding number of times any given peak must be applied to cause the rupture.

Heukelom[2] has reported on the fatigue properties of plain bitumens and has proposed that the fatigue strain $\epsilon_f(n)$ is given for large values of n as:

(16) $\epsilon_f(n) = 0.0385 \quad \bar{\epsilon}(1)\sqrt{S(T)}\ n^{-1/4}$

where S(T) is the constant load stiffness in kgf/cm^2 for a loading time equal to the period, T, of a complete cycle, and $\bar{\epsilon}(1)$ is the ultimate tensile strain corresponding to the stiffness S(T).

Of primary interest in traditional bituminous membrane systems is the fatigue characteristics of reinforcements likely to fail in fatigue **before** the bituminous coatings they carry.

According to RITC research, the following expression appears valid for wet process glass fibre tissues in the weight range of 50 to 80 g/m^2.

(17) $$\epsilon_f(n) = \bar{\epsilon}(1)\ \{1 - 0.25\ln(\cosh w)\}$$

where w = $\log_{10} n$
ln = natural logarithm to base e
$\bar{\epsilon}(1)$ = strain to break in one cycle
$\epsilon_f(n)$ = strain to break in n cycles

Expressions (16) and (17) are subject to scatter, and must be regarded as the expected or average fatigue characterisitcs only.

I hope to publish soon the corresponding characteristics for asbestos and organic fiber felts. A complication related to organic fibre felts, and to a lesser extent to asbestos felts, is that their mechanical properties greatly depend on the moisture content of felt.[3]

FATIGUE ENDURANCE OF SYSTEM

The succession of peak strains induced in a roof membrane, as for example given by Eq (15) for the proposed standard conditions, corresponds to an in situ fatigue test with the difference that the amplitude is not generally constant from cycle to cycle. It was noted above that, under standard conditions, the amplitude first diminishes owing to visco-elastic effects, and then rises due to the predominance of age hardening in the bitumen adhesive.

As there is no reason to suppose that the fatigue characteristics of a system in the field will differ materially from those under similar laboratory conditions, one may equate the fatigue characteristics with the envelope of peak strains generated by substrate movement to obtain an equation from which n may be calculated.

Since the induced strain is not generally constant from cycle to cycle, allowance for varying peaks may be made by introducing a usage factor based on the Palmgren-Miner relationship.

When the change in magnitude of each successive peak strain is small, one can show that the endurance is obtained more accurately by equating the fatigue endurance characteristic to the **cumulative** or moving average-induced strain $\epsilon_c(n)$, where

(18) $$\hat{\epsilon}_c(n) = \frac{1}{n}\sum_{1}^{n}\hat{\epsilon}(y)$$

When n is a large number, say, in excess of 50 cycles, Eq (18) approximates to the integral:

(19) $$\hat{\epsilon}_c(n) = \frac{1}{n}\int_1^{n+1}\hat{\epsilon}(y)\,dy$$

Substituting Eq (15) for $\hat{\epsilon}(y)$ in Eq (19), the cumulative average becomes:

(19a) $$\hat{\epsilon}_c(n) = \frac{1}{n}\int_1^{n+1}\beta g_o\sqrt{K(y)}\ \hat{Z}(y)\,dy = \beta g_o N(n), \text{ say}$$

where the function N(n) is fully determined by integration for the assumed conditions and for the given system properties. Note that y is a dummy variable for the purposes of integration.

The fatigue endurance of the system is given by the value of n which satisfies the identity.

(20) $$\epsilon_f(n) = \beta g_o N(n)$$

There will be a calculable endurance for the reinforcement, obtained by using, for example, Eq (17) for glass fibre tissue bases, and another for the normal bituminous coating obtained by using Eq (16).

Symbolically, one may write:

(21a) $\epsilon_{fb}(n) = \beta g_o N(n)$ for glass fibre felt, and

(21b) $\epsilon_{fc}(n) = \beta g_o N(n)$ for bituminous coating.

The practical endurance will be the lesser of the two values of n so calculated. However much one may improve the fatigue properties of a reinforcement, endurance will be limited by that of the coating bitumen.

Since both fatigue characteristics $\epsilon_f(n)$ and the function N(n) are ascertainable, one may conveniently rearrange terms to express the maximum joint movement g_o as some new function of n obtained by dividing $\epsilon_f(n)$ by N(n). It will also be convenient to separate out the strain $\bar{\epsilon}(1)$ and to look upon the fatigue characteristics as the product of $\bar{\epsilon}(1)$ and $\psi(n)$ such that $\epsilon_f(n) = \bar{\epsilon}(1)\,\psi(n)$ for each element of the membrane system.

The structural durability of the system is then characterised by the two identities:

(22a) $$\frac{\beta g_o}{\bar{\epsilon}_b(1)} = \frac{\psi_b(n)}{N(n)} = \phi_b(n)$$ say, for glass fibre felt, and

(22b) $$\frac{\beta g_o}{\bar{\epsilon}_c(1)} = \frac{\psi_c(n)}{N(n)} = \phi_c(n)$$ say, for the bituminous coating.

Figures 2 and 3 give the functions $\phi_b(n)$ and $\phi_c(n)$ for standard diurnal movements at 10°C with allowance for the probable exponential age hardening for a high PI bitumen (+ 6.3).

TREATMENT OF MULTILAYER SYSTEMS

A single layer bituminous system is primarily a uniform layer of bitumen reinforced by a **single** felt base embedded within it. A multilayer or built-up system is a generally thicker layer of bitumen reinforced by **two or more** superimposed bases, each separated from the other by bitumen to provide reinforcement at various depths within the composite membrane.

The strains developed in a multilayer system may be calculated from first principles by extending the basic single-layer model to include the additional reinforcement and then solving the related differential equations simultaneously.[4]

The superimposed felt reinforcements are not equally loaded by substrate movement. The share of load carried by a layer of reinforcement depends on its distance from the substrate and upon the relative elastic moduli of the several fabrics embedded in the bituminous mass.

The closer a reinforcing felt is to the substrate, the greater is its share of the load. The total rupture of a multilayer system can and frequently does occur by the separate failure of the reinforcement, almost as though the multilayer system consisted of as many sequentially activated single layer systems as there were layers of reinforcement, with due allowance made in each case for the distance separating the reinforcement from the common substrate beneath.

The greatest fatigue endurance is generally obtained by locating the most resistant reinforcement at the maximum feasible distance from the substrate, although this entails an acceptance of the possible sacrifice of the weaker felt bases beneath. The design principle adapted here is to dispose the reinforcement such as to delay for as long as possible the appearance of a fissure in the uppermost surface of the membrane.

The mechanical interaction of the several layers of embedded reinforcement may be reduced to a factor λ applied at the joint movement g, such that the latter becomes $g_o\lambda$ in all relevant expressions. The function depends on the relative moduli of the reinforcements and on the relative position of each layer of reinforcement above the substrate.

The order of fatigue endurance of a multilayer system may be calculated by an extension of the method developed for the single layer case. An outline of the procedure, which has generally proved satisfactory when applied to prediction of laboratory endurance tests, is given below for a two layer bituminous system.

The following convention is adopted:

(i) The felt reinforcements are numbered consecutively starting with No. 1 for the felt closest to the substrate.

(ii) λ_{12} = correction to base 1 when covered by base 2

λ_{21} = correction to base 2 when laid over base 1

λ_{2*} = correction to base 2 when base 1 is ruptured immediately beneath

(iii) Factors are calculated from the following:

$$\lambda_{12} = \frac{1+\sqrt{mr}}{\{1+m(1+r)+2\sqrt{mr}\}^{1/2}} \qquad (\lambda_{12} \leq 1)$$

$$\lambda_{21} = \frac{\sqrt{1+m}}{\{1+m(1+r)+2\sqrt{mr}\}^{1/2}} \qquad (\lambda_{21} \leq 1)$$

$$\lambda_{2*} = \frac{\sqrt{1+m}\,\{1+m(1+r)+2\sqrt{mr}\}^{1/2}}{1+m+\sqrt{mr}} \qquad (\lambda_{2*} \geq 1)$$

where $r = E_1/E_2$ = ratio of reinforcement moduli

$m = h_1/h_2$ = ratio of thickness of bituminous layers separating (i) base 1 from substrate and (ii) base 2 from base 1, respectively.

Case I: The fatigue endurance of base 1 exceeds that of base 2.

Where base 1 has a fatigue endurance greater than that of the superimposed layer, as, for example, when polyester or similar types of reinforcement are used, the action of base 1 will last for the life of the system.

The endurance of the system will then be the lesser of n_b or n_c where:

(23a) $$\phi_b(n_b) = \beta_2 g_o \lambda_{21}/\bar{\epsilon}_b \; : \; \beta_2 = 1/4\sqrt{3E_2(h_1+h_2)}$$

and

(23b) $$\phi_c(n_c) = \beta_2 g_o \lambda_{21}/\bar{\epsilon}_c$$

$\bar{\epsilon}_b$ and $\bar{\epsilon}_c$ are the ultimate strains on single application of load to rupture base 2 and the bituminous coating carried by base 2, respectively. The functions of n on the left hand side of the equations relate to base 2 and its external bituminous coating.

Case II: The fatigue endurance of base 1 is negligible

If substrate movement produces rupture of base 1 in the early phase of the membrane's service life the endurance of the system, which now depends entirely upon the integrity of base 2 and its coating, is given by the lesser of n_b and n_c where:

(24a) $$\phi_b(n) = \beta_2 g_o \lambda_{2*}/\bar{\epsilon}_b \; : \; \beta_2 = 1/4\sqrt{3hE_2(h_1+h_2)}$$

and

(24b) $$\phi_c(n) = \beta_2 g_o \lambda_{2*}/\bar{\epsilon}_c$$

Case III: The fatigue endurance of base 1 is non negligible but less than that of system: intermediate case

In this case, base 1 ruptures prematurely after n_1 cycles, where n_1 is less than n_b or n_c. The endurance of base 1 is obtained from:

(25) $$\phi_{b1}(n_1) = \beta_1 g_o \lambda_{12}/\bar{\epsilon}_{b1} \; : \; \beta_1 = 1/4\sqrt{3E_1 h_1}$$

The strain history for base 2 is now divided into two distinct phases. There is the early phase ($n<n_1$) during which base 1 provides structural assistance to base 2. This is followed by a later phase ($n>n_1$) when the ruptured base 1 accentuates the peak strain in base 2. The correction factor applicable to base 2 therefore changes from λ_{21} for $n<n_1$ to λ_{2*} for $n>n_1$.

Because the fatigue characteristic is equated to the cumulative average strain, the corresponding λ_{2*} now becomes dependent upon n and is weighted to allow for visco-clastic and age hardening effects.

The application of the principles established for the single layer case gives an adjusted factor:

(26) $$\lambda^1_{2*} = \lambda_{2*} - (\lambda_{2*} - \lambda_{21}) \frac{n_1 N(n_1)}{n N(n)}$$

where $n_1 \leq n$ and $\lambda_{21} \leq \lambda^1_{2*} < \lambda_{2*}$

The endurance of the system is given as before by the lesser of n_b and n_c where:

(27a) $$\phi_b(n_b) = \beta_2 g_o \lambda^1_{2*}/\bar{\epsilon}_{b2}$$

(27b) $$\phi_c(n_c) = \beta_2 g_o \lambda^1_{2*}/\bar{\epsilon}_{c2}$$

As λ^1_{2*} is itself a function of the unknown variable n, equations (27a) or (27b) are readily solved by a method of successive approximations.

COMMENTARY ON STRUCTURAL DURABILITY CALCULATIONS

The goal is to quantify structural endurance with due allowance for probable age hardening of bitumen, on the basis of a standard sinusoidal joint movement.

The only independent variable is the magnitude of the maximum joint movement, g_o. Other design parameters depend upon the properties and dimensions of the membrane materials.

In practice, product tolerances and unavoidable system variability may cause the endurance of two ostensibly identical membrane systems to differ appreciably. This feature is common to all engineering systems and is dealt with through appropriate factors of safety to allow for probable variability. It might be reasonable to design for a 50-year endurance to obtain a probable 20-year service life.

Perhaps a more useful application for the design procedure is to calculate **relative** endurance based on standard movement and aging conditions. Systems composed of similar materials could then be compared for structural endurance on an idealized but nonetheless rational common basis.

Note also that fatigue endurance and structural durability are essentially statistical concepts and do not purport to give anything other than an **expected** life for a **typical** system under **average** conditions. Yet that alone is more than is presently available and constitutes a step in the right direction.

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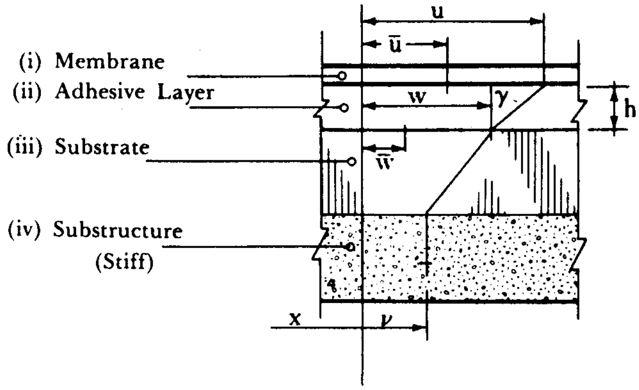
APPENDIX I
SYMBOLS AND NOMENCLATURE

The principal sybols are generally listed in order of appearance in the text of Part II.

$f(xt)$	=	any real function of t and x
$f^*(xp)$	=	Carson-Laplace transform of f(t), where $f^*(p) = p\int_0^{\infty} f(xt)\exp(-pt)\,dt$
$F(t)$	=	Constant load flow function for bitumen at constant temperature
$S(t)$	=	Constant load stiffness = $1/F(t)$
J	=	Instantaneous elastic compliance of bitumen, (cm^2/kgf)
K	=	Bitumen stiffness per unit load time (delayed elastic component of deformation (kgf/cm^2)
α	=	Power index related to PI of bitumen as given in $F(t) = t^{\alpha}/K$.
T, t	=	Loading time (sec)
τ	=	Bitumen shear stress, (kgf/cm^2)
γ	=	Bitumen angular distortion
$r(t)$	=	Relaxation function for bitumen
E	=	Elastic modulus of membrane or reinforcement (kgf/cm)
M	=	Shear rigidity of substrate (kgf/cm^3)
h	=	Total thickness of bituminous layer between reinforcement and substrate (cm)
u, w, ν	=	System displacements (see Fig. 1)
$\bar{u}, \bar{w}$	=	Unrestrained temperature displacements (see Fig. 1)
ϵ	=	Strain in membrane or reinforcement
k_m, k_s	=	Coefficients of thermal expansion for deg °C
θ	=	Temperature change, deg°C
g	=	Joint opening function g(b), (cm)
g_o	=	Maximum joint opening, (cm)
$\epsilon_f(n)$	=	Fatigue characteristics of an element of membrane
$\bar{\epsilon}$ or $\bar{\epsilon}(1)$	=	Ultimate strain on a single load application
$\hat{\epsilon}_c(n)$	=	'Moving average' envelope of peak tensile strain in reinforcement
$\psi(n)$	=	$\epsilon_f(n) \div \bar{\epsilon}(1)$
$N(n)$	=	Moving average strain factor with allowance for age hardening effects in bitumen
$\phi(n)$.	=	$\psi(n)/N(n)$

APPENDIX II
RUBEROID INTERCOMPANY TECHNICAL COMMITTEE MEMBERS
ALPHABETIC LIST

Asfaltex SA	Spain
Bitroid Ltd	South Africa
Cindu Key & Kramer NV	Netherland
FEREM SA	France
Hotaco A/S	Denmark
Lemminkainen Oy	Finland
Prodotti Gamma Ruberoid	Italy
Ruberoid Ltd	United Kingdom
Ruberoidwerke AG	Germany



Any Point at Co-ord x

Substructure movement results in:

$u(x)$ = displacement of membrane
$w(x)$ = displacement of substrate at adhesive interface
$\nu(x)$ = displacement of substructure

Temperature Effects: Where applicable

$\bar{u}(x)$ = unrestrained displacement of membrane
$\bar{w}(x)$ = unrestrained displacement of substrate

PART II
FIG. 1.