

*THE THEORY OF MICRO-METEORITES.\* PART II. IN  
HETEROTHERMAL ATMOSPHERES*

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*Introduction.*—In a previous paper<sup>1</sup> the writer presented a theory for the passage of a small meteoritic body through the earth's atmosphere without damage by melting or vaporization. The theory, which derives the maximum ratio of mass to effective surface area for a micro-meteorite of a given initial velocity, is valid for an atmosphere of constant temperature and mean molecular weight but depends upon certain assumptions. The present paper extends the theory to more general atmospheric circumstances and also explores the tenability of the basic assumptions. For convenience, the previous notation is utilized while references and equations of the first paper are identified by asterisks.

1. *The Solution in an Atmosphere of Constant Temperature Gradient.*—At altitudes in the neighborhood of 100 kilometers, most investigators at present tend to adopt an atmospheric temperature increase with altitude.<sup>2</sup> Little is certainly known about the mean molecular weight at these altitudes but a constant value will probably be a fair assumption. The dissociation of O<sub>2</sub> probably occurs near 100 kilometers altitude but its effect on the density may be absorbed in the assumed temperature gradient for constant molecular weight, as may the slight effect arising from the decrease of gravity with height.

It is, therefore, of great interest to repeat the solution for the maximum ratio of mass to surface area, applying to micro-meteorites in an atmosphere with a constant temperature gradient. Let the atmospheric temperature,  $T_a$ , vary with height,  $h$ , according to the relation

$$T_a = T_1 + \nu h, \quad (1)$$

where  $T_1$  is the temperature at a height,  $h = 0$  and where  $\nu$  remains constant.

The atmospheric density then can be expressed by the relation<sup>3</sup>

$$\rho = \rho_1 \left( 1 + \frac{\nu h}{T_1} \right)^{-(1+q/\nu)}, \quad (2)$$

where  $\rho_1$  is the density at  $h = 0$ .

The quantity  $q$  is defined by

$$q = \mu g/k, \quad (3)$$

where  $\mu$  is the mean molecular weight of the air,  $g$  is the attraction of earth gravity corrected for altitude and  $k$  is the Boltzmann Constant. The logarithmic density gradient,  $b$ , used previously is equal to  $(q + \nu)/T_a$ .

With our previously adopted assumptions that the drag coefficient,  $D$ , accommodation coefficient,  $\alpha$ , and emissivity,  $\beta$ , are constants we may substitute  $\rho$  from equation (2) for its isothermal value in equation (14\*) and integrate the velocity differential to obtain

$$\log (V/V_\infty) = - \frac{ADT_1\rho_1}{2mq \cos Z} \left(1 + \frac{\nu h}{T_1}\right)^{-q/\nu} \quad (4)$$

Equation (10\*), giving the surface temperature of the micro-meteorite, now becomes

$$T_s^4 - T_0^4 = \frac{\alpha A \rho_1 V^3}{2\beta B \sigma} \left(1 + \frac{\nu h}{T_1}\right)^{-(1+q/\nu)}, \quad (5)$$

where we again neglect the heat capacity.

Upon eliminating the terms in  $h$  from equations (4) and (5), we express the temperature of the micro-meteorite in terms of the velocity, independently of height or density, as follows:

$$T_s^4 - T_0^4 = \frac{\alpha V^3}{\beta B \sigma} \left(\frac{2}{A\rho_1}\right)^{\nu/q} \left[- \frac{mq \cos Z}{DT_1} \log (V/V_\infty)\right]^{(\nu+q)/q} \quad (6)$$

By equating to zero the derivative with respect to  $V$  of the right member of equation (6) we find that the maximum temperature of the meteoroid,  $T_m$  occurs at the critical velocity,  $V_c$ , given by

$$\log (V_c/V_\infty) = - \frac{\nu + q}{3q} \quad (7)$$

This value of the velocity is now substituted into equation (6),  $A$  is equated to  $A_1$  and expressed in terms of  $B$  by equation (2\*) and the ratio of mass to surface area for the micro-meteorite is obtained in the form

$$\frac{m}{B} = \frac{3eDT_1}{(\nu + q) \cos Z} \left(\frac{\rho_1}{8}\right)^{\nu/(\nu+q)} \left[\frac{\beta\sigma(T_m^4 - T_0^4)}{\alpha V_\infty^3}\right]^{q/(\nu+q)} \quad (8)$$

The reader will note that when  $\nu = 0$ , corresponding to an atmosphere of uniform temperature and molecular weight, equation (8) reduces to equation (18a\*), since  $b = q/T_1$ . Again the radius of a spherical meteoroid of density,  $\rho_s$ , is  $s = (3m)/(\rho_s B)$ .

For a temperature gradient of  $+4^\circ$  per kilometer and  $V_\infty = 23.1$  km./sec., equation (8) yields a value of  $m/B$  some 10% greater than equation (18a\*) for an identical atmospheric density and density gradient at the critical velocity in the isothermal case. Since these solutions are in such

close numerical agreement, we may trust equation (8) to give a rather precise result for most problems of micro-meteorites.

2. *The Solution in the General Case.*—When the physical data justify a more precise theory for micro-meteorites than that given in the previous section, a numerical integration will be required generally. The atmospheric temperature will certainly vary with height in some complex fashion while its mean molecular weight may also vary appreciably. The drag coefficient and the emissivity may depend upon the velocity and temperature of the micro-meteorite and possibly upon the air density. In addition the heat capacity,  $C_s$ , may be appreciable and will vary with  $T_s$ .

Before going on to the most general situation, let us investigate the error introduced by neglecting the heat capacity in the previous solutions. Beginning with equation (7\*), the temperature differential can be related to the velocity differential by means of equation (13\*), (14\*) and (15\*) so that the time, height and atmospheric density are explicitly eliminated. The temperature derivative with respect to the velocity is

$$\frac{dT_s}{dV} = -uV - \frac{w(T_s^4 - T_0^4)}{V^2 \log(V/V_\infty)}, \quad (9)$$

where the constants,  $u$  and  $w$ , are given by

$$u = \alpha/(C_s D) \quad (10)$$

and

$$w = \beta B \sigma / (b m C_s \cos Z). \quad (11)$$

Our previous assumption in neglecting the heat capacity was derived by equating the right member of equation (9) to zero. The resulting equation (16\*) is rewritten:

$$T_s^4 - T_0^4 = -uV^3(\log V/V_\infty)/w. \quad (12)$$

We may now derive a second approximation to the temperature by substituting the derivative of equation (12) into equation (9). This operation leads to the expression

$$T_s^4 - T_0^4 = -\frac{u}{w} V^3 \log(V/V_\infty) \left[ 1 - \frac{V + 3V \log(V/V_\infty)}{4wT_s^3} \right]. \quad (13)$$

If we now differentiate equation (13) and set  $dT_s/dV = 0$  the result will be a second approximation to the velocity-temperature relation at maximum temperature,  $T_m$ . The equation is written more simply with the first order auxiliaries  $x$  and  $y$  where

$$x = V_c / (4wT_m^3), \quad (14)$$

and

$$y = 1 + 3 \log (V_c/V_\infty). \quad (15)$$

After dropping non-zero coefficients, we find the relation at maximum temperature to be

$$y(1 - xy) = x(y - 1) (1 + y/3). \quad (16)$$

If we drop second-order terms in  $x$  and  $y$ , equation (16) simplifies to

$$y = -x = -\frac{V_\infty e^{-1/3} e^{y/3}}{4wT_m^3} = -\frac{V_\infty e^{-1/3}}{4wT_m^3}. \quad (17)$$

Hence, under the same conditions,

$$\log (V_c/V_\infty) = (y - 1)/3, \quad (18)$$

and

$$V_c^3 = V_\infty^3(1 + y)/e. \quad (19)$$

Thus  $V_c^3$  is reduced by a first-order term ( $-y$ , which is positive). A substitution of the corrected value of  $V_c$  into equation (13) to evaluate  $T_m$ , however, leaves no first-order correction terms. The original value of  $T_m^4$  is unchanged except for second and higher-order terms ( $1 - 2y^2 + \dots$ ). It is clear that no successive approximations can introduce a first-order term into  $T_m^4$ . We must conclude, therefore, that the inclusion of the heat-capacity effect into our solution for the maximum temperature of the micro-meteorite reduces by a first-order term the velocity at which the maximum occurs, and correspondingly lowers the atmospheric height and increases the density; but it does not change the maximum temperature by a first-order term.

The value of the correction term,  $y$ , expressed in terms of the original physical quantities involving  $m/B$  from equation (18a\*), may be written

$$y = -3e^{2/3} C_s D T_m (1 - T_0^3/T_m^3)/(4\alpha V_\infty^2). \quad (20)$$

The quantity  $y$  is a maximum at the minimum value of  $V_\infty$ , 11.2 km./sec. We may adopt an extremely high value of  $C_s = 0.3$  cal./g. for irons or stones ( $C_s = 1.3 \times 10^7$  ergs/g.),  $\alpha = 0.9$ ,  $D = 2.2$ ,  $T_m = 1600^\circ\text{K.}$ , and neglect  $T_0$ . The resulting value of  $y$  appears to be a maximum for any micro-meteorites traversing the earth's atmosphere undamaged. This value is  $y = -0.063$ . Since the correction term to  $T_m^4 - T_0^4$  is  $1 - 2y^2, \dots$ , the maximum correction is  $-0.008$ , less than 1%.

Of critical importance is the fact that the  $\cos Z$  term cancels out of  $y$  in equation (20). This cancellation means that the correction term is independent of the angle at which the micro-meteorite strikes the atmosphere. Even though relatively massive bodies may be stopped without melting when  $\cos Z$  is quite small, the effect of heat capacity is no more serious than

for a much smaller body striking vertically. As  $\cos Z$  decreases, the time available for equilibrium increases in such a fashion as to cancel the correction term.

The question of the mean free path of air molecules compared to the dimensions of the micro-meteorites must be investigated. Since the momentarily trapped air molecules leave the micro-meteorite at the thermal velocity,  $v_T$ , of the surface, their density near the surface tends to increase above the ambient air density by a factor of the order of  $V/v_T$ . The impinging air molecules must pass through this haze of escaping gas for a distance comparable to the diameter of the body. If the mean free path of air molecules in the region surrounding the micro-meteorite near the time of maximum surface temperature is greater by a factor of  $V/v_T$  than the diameter of the body, we may be reasonably certain that the present theory requires no correction for this effect. There is every reason to believe that the mean free paths of the air molecules at the high velocities here encountered will be greater than those calculated at normal temperature. Hence, if our limit is a safe one for air at normal temperature, it will certainly be safe under the conditions of the present theory.

An inspection of table 1 in the next section shows that the maximum diameter of an undamaged micro-meteorite at vertical incidence velocity is of the order of 0.01 cm. or smaller. The maximum air density at maximum surface temperature is of the order of  $6 \times 10^{-9}$  g./cm.<sup>3</sup> At normal temperature the mean free paths of air molecules at this density is of the order of 1 cm. Hence the mean free path exceeds the diameter of the micro-meteorite by a factor of approximately one hundred times. The ratio  $V/v_T$  is only of the order of ten. Hence the effect of mean free path may be neglected unless  $\cos Z < 1/10$ , in which case  $2s$  may be comparable to the standard mean free path. In fact the approximation is rather good for radiants to about  $1^\circ$  of the horizon ( $Z = 89^\circ$ ) or even farther, because of the effective increase of mean free path with velocity and because the effect of molecular encounters changes the theory slowly as the mean free path decreases.

Since  $\rho_m$  and  $s$  vary roughly in the same fashion with  $V_\infty$ , and since  $s$  varies as  $\sec Z$ , the problem of mean free path is a function of  $Z$  only. For radiants below an altitude of about  $1^\circ$  the calculated maximum dimensions are somewhat underestimated in the present formulae because the accommodation coefficient may be reduced by the shielding effect.

The time-lag in heat transfer from the front to back surface of the micro-meteorite may now be investigated. Suppose that the micro-meteorite does not rotate. A slight development of the "cold wave" approximation of heat transfer as used by L. R. Ingersoll and O. J. Zobel<sup>4</sup> can be applied to this case. A temperature differential over a distance  $L$  will be reduced to  $1/e$  of its value in a time  $\pi L^2/(2H^2)$ , where  $H^2$  is the heat diffusivity of the

material. For ordinary stone  $H^2 \cong 0.005$  cm.<sup>2</sup>/sec. For solid metals its value is much larger. With  $H^2 \cong 0.005$  and  $L = 0.01$  cm. (the largest value of  $2s$  for  $\cos Z = 1$ ), the time is 0.03 sec. Unless the meteoric material is a much poorer conductor than ordinary stone, the lag in heat transfer will not be serious for the largest micro-meteorites at vertical incidence. At lower angles of incidence the time of travel increases as  $\sec Z$ , as does the dimension of the body. Hence the time lag increases as  $\sec Z$ . The lag begins to introduce appreciable error for  $Z > 87^\circ$ . The effect will reduce the radiation rate somewhat below its assumed value in the present theory. The calculated critical dimension of a non-rotating micro-meteorite will be overestimated at the lowest velocities in the range  $87^\circ < Z < 89^\circ$ . If the body is rotating rapidly the error introduced by heat lag will usually be small, even to  $Z = 89^\circ$ .

The present theory, however, underestimates the critical dimensions of micro-meteorites at extremely large zenith angles because the curvature of the Earth is neglected. In no case can the curvature of the path completely compensate for this effect in view of the fact that the critical velocity is  $0.716 V_\infty$ , while the velocity in a circular orbit is 0.707 times the minimum velocity of fall from infinity. We may roughly define the limits set by curvature in the following fashion: while the micro-meteorite is falling through a vertical distance  $1/b$ , in which the air density increases by a factor  $e$ , the change in height introduced by curvature,  $h$ , must not exceed  $\delta/b$ , where  $\delta$  is small. The horizontal distance,  $S$ , then becomes

$$S = (\tan Z)/b. \quad (21)$$

The corresponding change in height,  $\Delta h$ , arising from curvature over the horizontal distance,  $L$ , is given to sufficient accuracy by

$$\Delta h = S^2/2R, \quad (22)$$

where  $R$  is the radius of the Earth.

If we substitute  $\Delta h = \delta/b$  in equation (22) and substitute  $S$  from equation (21) we find

$$\tan^2 Z = 2bRS. \quad (23)$$

In an atmosphere of constant temperature gradient, the value of  $b$  becomes

$$b = \frac{q + \nu}{T_1} \left( \frac{\rho_1}{\rho} \right)^{\nu/(q + \nu)}. \quad (24)$$

Hence the limiting value of zenith distance,  $Z_{\max.}$ , from equation (23) can be shown to be given by

$$\tan^2 Z_{\max.} = \frac{2R(q + \nu)\delta}{T_1} \left( \frac{\rho_1}{\rho} \right)^{\nu/(q + \nu)}. \quad (25)$$

For  $\delta = 1/100$ , or 1% accuracy, the values of  $Z_{\max}$  lie in the range of  $70^\circ$  to  $77^\circ$  for  $V_\infty$  having minimum and maximum values, respectively, applying to solar-system micro-meteorites. For  $\delta = 1/10$ , the corresponding range in  $Z_{\max}$  is  $84^\circ$  to  $86^\circ$  (see next section).

As a consequence of the above several considerations we may neglect effects arising from heat capacity, secondary encounter processes and heat transfer in the above explicit theory of undamaged micro-meteorites to limits of zenith angle at which curvature of the earth becomes appreciable. The precision, within the limits of the assumptions, is within 1% for all micro-meteorites striking at Zenith angle less than roughly 70% and within about 10% for zenith angles between 70 and 84%. The theory probably represents the general order of magnitude to a zenith angle of about  $87^\circ$ , when effects of earth curvature become serious for all velocities, while the incomplete heat transfer may introduce further errors at low velocities.

Since Earth curvature and curvature of path can be allowed for in the general theory where numerical integration is used, this theory can be trusted to high precision to a zenith angle of approximately  $87^\circ$  for the minimum velocity. At velocities exceeding 20 km./sec., the theory should be reliable to a zenith angle of about  $89^\circ$  or perhaps a little greater.

The fundamental equations to be solved numerically are:

$$\frac{dV}{dt} = -\frac{AD\rho V^2}{2m}, \quad (11^*)$$

and

$$T_s^4 - T_0^4 = \frac{\alpha A \rho V^3}{2\beta B \sigma}, \quad (26)$$

and

$$dt = -dh/(V \cos z), \quad (13^*)$$

in case Earth curvature may be neglected.

Otherwise  $h$  and  $t$  must be related by more accurate expressions involving earth curvature and the effect of gravity introduced in the motion. The atmospheric density,  $\rho$ , must be adopted as a function of height,  $h$ , from some standard atmosphere.

Until a more detailed theory is adopted, such as that by Miss Heineman, the drag coefficient,  $D$ , may be determined by the expression

$$D = 2 + \frac{8}{9V} \left( \frac{8k_1 T_s}{\pi \mu_1} \right)^{1/2}. \quad (27)$$

In the above equations (11\*, 26, 13\*, 27) only  $m$ ,  $T_0$ ,  $k_1$ , and  $\sigma$  may be taken as strictly constant in the most general case. The numerical integration, however, still remains a relatively simple type of operation.

The author has met with little success in calculating the degree of surface deterioration that may occur to a micro-meteorite at higher velocities when atmospheric molecules impinge on the surface with energies of several hundred electron volts. Near the lower limit of velocity, 11.2 km./sec., the energies are so small that the deterioration should not be appreciable. Near the upper limit of solar-system velocities, 72 km./sec., the atmospheric molecules must penetrate deeply below the surface, dissociate and damage the lattice structure of the solid. Since the total air mass encountered before the velocity becomes greatly reduced is comparable to the mass of the micro-meteorite, this damage may be considerable. A crude estimate places the critical velocity in the neighborhood of 30 km./sec. Above this velocity the damage probably becomes appreciable and near

TABLE 1  
NUMERICAL VALUES FOR MICRO-METEORITES

$V_{\infty}$ , KM./SEC.	$D$	$\rho$ (MAX. $T$ ), G./CM. <sup>3</sup>	NACA MASS*/ SURFACE, G./CM. <sup>2</sup> $\times 10^4$	V-2 MASS*/ SURFACE, G./CM. <sup>2</sup> $\times 10^4$
11.3	2.09	$5.6 \times 10^{-9}$	41.4	24.3
15.0	2.07	$2.4 \times 10^{-9}$	19.2	11.8
20.0	2.05	$1.0 \times 10^{-9}$	8.8	5.6
23.1	2.04	$6.5 \times 10^{-10}$	6.0	3.9
25.0	2.04	$5.2 \times 10^{-10}$	4.8	3.2
30.0	2.03	$3.0 \times 10^{-10}$	2.94	1.99
40.0	2.02	$1.3 \times 10^{-10}$	1.36	0.96
50.0	2.02	$6.5 \times 10^{-11}$	0.74	0.54
60.0	2.02	$3.7 \times 10^{-11}$	0.46	0.34
70.0	2.01	$2.4 \times 10^{-11}$	0.30	0.23

\* The tabulated quantity is radius in microns for a spherical body of density  $3(s = 3m/B\rho s)$  at vertical incidence. At other angles of incidence the quantities should be multiplied by secant zenith distance of the radiant. See equation (18c\*), *et seq.*, for dimensions of non-spherical particles.

70 km./sec. may become quite destructive. At these velocities, however, the maximum dimensions of micro-meteorites are the order of only one micron (table 1, above), rather small for easy study at the earth's surface.

3. *Numerical Results.*—The atmospheric density,  $\rho_m$ , at the maximum surface temperature of the micro-meteorite is given by equation (20\*). Although this equation is derived on the assumption of an atmosphere of constant temperature, molecular weight and gravity, nevertheless, it gives the correct order of magnitude for  $\rho_m$ . The constants,  $\alpha$ , accommodation coefficient and,  $\beta$ , emissivity, we have already adopted as nearly unity and, for lack of better information, equal. If we adopt a maximum temperature,  $T_m$ , equal to 1600°K., and  $T_0 = 300^\circ\text{K.}$  then equation (20\*) becomes numerically,



$$\rho_m = 8.1 \times 10^9 / V_\infty^3, \quad (28)$$

where c. g. s. units are used.

Numerical values of  $\rho_m$  are given in the third column of table 1 for various values of  $V_\infty$  in the first column.

The value of the drag coefficient,  $D$ , is determined from equation (22\*) with the substitution of  $V_\infty$  for  $V$ . A mean molecular weight of 26.5 and the maximum temperature,  $T_m$ , are used in calculating,  $D$ . The resulting numerical equation is

$$D = 2 + 1.0 \times 10^6 / V_\infty \quad (29)$$

where  $V_\infty$  is expressed in cm./sec.

Numerical values of  $D$  are given in the second column of table 1.

The limiting ratio of mass to total surface area,  $m/B$ , for micro-meteorites is calculated only from equation (8), in an atmosphere of constant temperature gradient. Vertical incidence is assumed so that  $\cos Z = 1$ . The remaining constants depend upon the atmospheric conditions. Two solutions are given, one based upon the Tentative Standard Atmosphere of the National Advisory Committee for Aeronautics<sup>5</sup> and the second based upon atmospheric results from the V-2 rocket firings.

The data adopted as a close approximation to the NACA atmosphere are:  $h = 0$  at 83 km. altitude,  $\rho_1 = 3.32 \times 10^{-8}$  g./cm.<sup>3</sup>,  $T_1 = 240^\circ\text{K}$ .,  $\mu = 26.5$ ,  $g = 950$  c. g. s.,  $q = 3.05 \times 10^{-4}$  °C./cm. and  $\nu = 3.65 \times 10^{-5}$  °C./cm. The temperature gradient is, therefore, positive with height at a rate of 3.65 °C./km., beginning at 240°K. at an altitude of 83 km. The resulting numerical form of equation (8) is

$$m/B = 3.27 \times 10^{13} D \sec Z V_\infty^{-2.68}. \quad (30a)$$

Numerical results from equation (30a) for  $\cos Z = 1$  are given in the fourth column of table 1.

Typical results for atmospheric density by measures from V-2 rockets are given by E. Durand<sup>6</sup> of the Naval Research Laboratory. Adopted are:  $h = 0$  at 100 km. altitude,  $\rho_1 = 1.00 \times 10^{-9}$  g./cm.<sup>3</sup>,  $T_1 = 223^\circ$ ,  $\mu = 26.5$ ,  $g = 950$  c. g. s.,  $q = 3.05 \times 10^{-4}$  °C./cm. and  $\nu = 5.6 \times 10^{-5}$  °C./cm. Hence the temperature, beginning at 223° at 100 km., increases with height at a rate of 5.6° C./km. The numerical form of equation (30) is

$$m/B = 2.55 \times 10^{12} D \sec Z V_\infty^{-2.53}. \quad (30b)$$

This equation leads to the results given in the fifth column of table 1. A comparison of columns four and five shows that the V-2 atmosphere leads to smaller values of  $m/B$  than the NACA atmosphere in a ratio of about 2/3, little dependent upon velocity. This difference arises largely from the difference in logarithmic density gradient between the two atmospheres

at corresponding values of the density. The critical heights range from just below 100 km. to roughly 140 km.

The calculations of the limits of zenith distance at which the curvature of the Earth introduces appreciable error, given by equation (25) and presented near the end of the previous section, were derived from the V-2 atmosphere. This atmosphere gives less favorable limits than the NACA atmosphere.

Quantities in table 1 are also presented specifically for  $V_{\infty} = 23.1$  km./sec., corresponding to the Giacobinid meteor shower. In the Washington, D. C. area,  $\cos Z$  was about 0.45 for this shower at the time of maximum activity. Hence for undamaged iron micro-meteorites of density 7.7, the maximum spherical radii should have been 5.2 microns for the NACA atmosphere and 3.4 microns for the V-2 atmosphere. For stony-irons of density 4.0 these values become 10.0 and 6.5 microns, respectively. For long cylinders of circular cross-section, the minor diameter is  $4/3$  the spherical radius, giving a predicted range from 4.2 to 13.3 microns, the range depending upon uncertainties in the atmosphere and in the density of the micro-meteorites.

Coincident with the shower, allowing for time of fall, H. E. Landsberg<sup>7</sup> found at Mt. Weather, Va., "wedge-shaped opaque" magnetic particles of dimensions  $40 \times 5$ ,  $40 \times 5$  and  $100 \times 4$  microns. These he attributed to the shower. The agreement of the minor dimension with the present predictions is entirely satisfactory. He also found 3 "round" particles of diameter 5 to 10 microns, again in agreement with this theory. On the other hand, he found a few "round" particles of diameters from 20 to 40 microns. Although these are too large for the present theory, there is no reason to doubt that larger bodies may have been melted and stopped with minor surface vaporization, which would certainly tend to smooth the surface—in keeping with Landsberg's observations. Opik<sup>8</sup> discusses the formation of droplets in meteoroids in considerable detail.

Landsberg estimates roughly a rate of fall of  $3^{1/2}$  miles per day for the particles of diameter 4 to 10 microns, 13 miles per day for those of diameter 20–30 microns and 100 miles per day for the one of diameter of 60 microns. The expected times of fall would be quite variable for the smaller particles because of atmospheric turbulence.

Generally, then, the present theory is in good agreement with Landsberg's observations. The critical dimensions here calculated are of most value in predicting a dimension that divides irregular rough-surfaced particles from larger ones that have been melted and partially vaporized in passing through the atmosphere.

It is now clear that the dependence of critical particle size on assumed atmospheric conditions is too small to make the study of micro-meteorites valuable at this time as a measure of atmospheric density gradient. Nevertheless, the minor dimensions of the largest rough irregular wedge-shaped

pieces observed by Landsberg fall closer to the prediction of the V-2 atmosphere than the NACA atmosphere. It is difficult to know how to weight the significance of these results, particularly in view of uncertainties in the physical characteristics of micro-meteorites.

Generally the present theory may be most useful in the following respects:

(a) It provides theoretical evidence that micro-meteorites can fall undamaged, hence gives impetus to their collection and study, which should lead to measures of space densities for small particles.

(b) It can aid in distinguishing real micro-meteorites from terrestrial objects.

(c) It can aid in identifying micro-meteorites from two nearly coincident meteor showers.

(d) It can assist in determining the original velocity of recognized micro-meteorites.

(e) It may help provide evidence concerning the final dimensions of particles producing meteors.

The collection and laboratory study of micro-meteorites from meteoric streams can be invaluable in establishing the nature of interplanetary material, specifically cometary material. If the writer's present working hypothesis is correct, viz., that comets were formed at great solar distances by the condensation of gas at very low temperatures, then micro-meteorites from meteor showers may represent the only laboratory samples of typical or atypical interstellar solids. Ordinary meteorites appear to arise from a different source.<sup>9</sup>

On a purely speculative level is the possibility of detecting evidence for past solar-system catastrophes. If a minor or major planetary disruption actually occurred in recent astronomical time, as suggested by C. A. Bauer<sup>10</sup> to account for the meteoritic helium contents, evidence for the concomitant micro-meteorites may conceivably be found in the chalk beds of the Cretaceous Period or in other geological formations.

Furthermore, there is the possibility that micro-meteorites in deep oceanic sediments (as suggested by the *Challenger Expedition*), in glacial snows or even in geological formations (such as chalk beds) may provide a record of past astronomical activities.

The writer earnestly hopes that this paper may encourage the collection and study of micro-meteorites. Current work on this problem is being conducted by D. K. Norris and F. S. Hogg,<sup>11</sup> while J. D. Buddhue<sup>12</sup> has recently summarized his and others' observation data on meteoritic dust in general.

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<sup>1</sup> Whipple, F. L., these PROCEEDINGS 36, 687-695, 1950.

<sup>2</sup> See, e.g., Mitra, S. K., *The Upper Atmosphere*, ch. III, The Royal Asiatic Society of Bengal, 1948.

<sup>3</sup> See, e.g., Mitra, S. K., *loc. cit.*, p. 6.

<sup>4</sup> *Mathematical Theory of Heat Conduction*, Ginn and Co., 1913, p. 40.

<sup>5</sup> Warfield, C. N., NACA, Tech. Note, No. 1200, Langley Field, Jan., 1947.

<sup>6</sup> *The Atmospheres of the Earth & Planets*, Univ. of Chicago Press, 1948, p. 142.

<sup>7</sup> *Pop. Ast.*, **55**, 322 (1947).

<sup>8</sup> *Pub. Univ. Tartu*, **29**, No. 5, 51 (1937).

<sup>9</sup> See, e.g., Brown, H., and Patterson, C., *J. Geol.*, **56**, 85 (1948).

<sup>10</sup> *Phys. Rev.*, **74**, 501 (1948).

<sup>11</sup> Reported at the June 1949 meeting of the Am. Ast. Soc.

<sup>12</sup> Univ. N. Mex. Pub. in Meteoritics, No. 2, 1950.

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## MULTIMOLECULAR ADSORPTION OF HORSE HEART METMYOGLOBIN ONTO FILMS OF BARIUM STEARATE\*

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In a previous paper<sup>1</sup> it was shown that some protein solutions, when adsorbed onto films of barium stearate, followed the classical Langmuir adsorption isotherm. The thicknesses of these unimolecular layers were calculated by using the Langmuir equation, and it was shown that these values for human and horse hemoglobin and for bovine serum albumin agreed with values in the literature for the thicknesses of these molecules.

It was felt advisable to test this technique on another protein of size and molecular weight different from the above proteins. Myoglobin was chosen because its dimensions have been recently determined by x-ray diffraction.<sup>2</sup>

*Experimental.*—The technique was adequately described in the previous paper. The thicknesses of the metmyoglobin layers adsorbed onto metallic slides covered with an optical gauge of barium stearate were determined on the ellipsometer, an optical instrument which measures the ellipticity of polarized light reflected from a metallic surface.

The metmyoglobin solutions were prepared from fresh horse heart by repeated precipitations from strong phosphate buffers. The Beckman spectrophotometer measurement of a carbonmonoxylated solution did not show the presence of any contaminating hemoglobin. The electrophoresis of a metmyoglobin solution showed that two components were present, the smaller component representing 10% of the total area. An electrophoresis of a carbonmonoxymyoglobin solution showed that the smaller component was present in one-half its concentration in the metmyoglobin